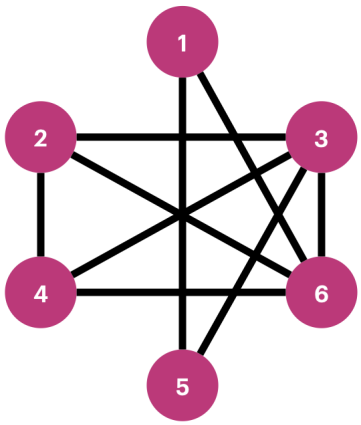
Grade 9/10 Math Circles

November 8, 2023

Graph Theory - Problem Set

Graph Review

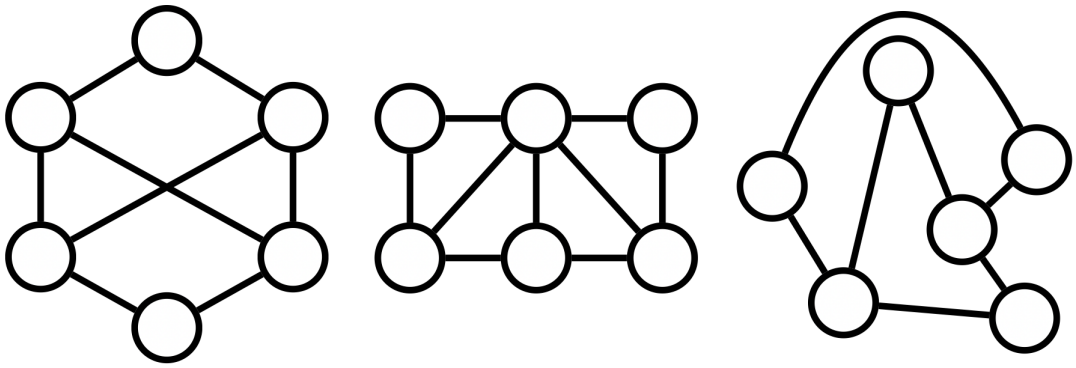


Graph A

1. For Graph A:
   (a) Find the neighbours of vertex 1 and vertex 6
   (b) Find a walk from vertex 1 to 4 and a path from 2 to 5
   (c) Find a cycle, a subgraph and a spanning tree
   (d) Find a 4-colouring
2. Draw a tree (the graph kind!)
3. In this problem we look at a weighted graph:
   (a) Draw a weighted graph with 8 vertices, labelled 1-8.
   (b) Find a path from vertex 2 to 6.
   (c) What is the weight of the path you found?

Graph Colouring

1. Draw a bipartite graph.
2. Which of the following graphs are bipartite?



3. Find a colouring for each of the above graphs with the minimum number of colours.
4. Find a condition that would make a graph **NOT**:
   (a) 3-colourable
   (b) 4-colourable
   (c) **Challenge:** k-colourable

Timetabling

1. This problem is called "Aircraft Scheduling" and comes from the 2019 BCC. Use graph colouring to solve this problem:

   When an aircraft lands at an airport, it is assigned a designated airspace called a *corridor*. By ensuring that flights with similar landing times are in different corridors, air traffic controllers can help to avoid accidents.

   At the Bebrasland airport, two aircraft cannot have the same corridor if their landing times are within 15 minutes of each other.

   You are the Air Traffic Controller at the airport and your job is to assign corridors for the flights that are due to land at the times shown in the table.

Flight	Time
A	7:00am
B	7:21am
C	7:20am
D	7:18am
E	7:03am
F	7:12am

   What is the minimum number of corridors needed to ensure that the flights in the above table are assigned corridors according to the rules at the Bebrasland airport?

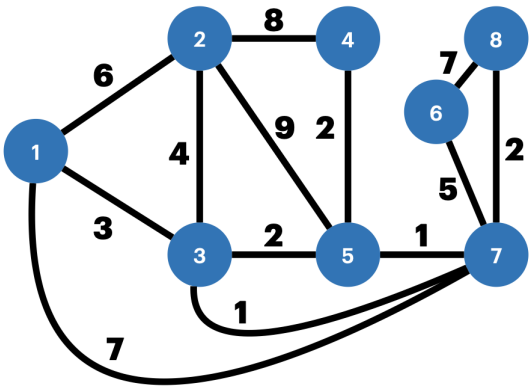
2. **Challenge:** Think of a real-life example of where graph colouring might be used (like our example with aircraft or class scheduling). Create a practice problem for this example and have a friend try solving the problem using graph colouring.

Greedy Colouring Algorithm

1. Find a colouring for Graph B using the greedy colouring algorithm.
2. Does your colouring use the minimum amount of colours? If not, see if you can find a colouring which uses the fewest colours possible.
3. **Challenge:** Create a graph, colour order and vertex order such that when the greedy colouring algorithm is applied (using the given orders), we do not achieve a minimum colouring.

Prim's Algorithm

1. Find an MST of Graph B using Prim's Algorithm.



Graph B